Exam Optimization of Business Processes
23 May 2011

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions count equally.
The use of a calculator and a dictionary are allowed.
A table with the Poisson distribution is attached.

1. Consider a model for the waiting times of walk-ins for a CT scanner. Scan times are approximately normally distributed. There are two types of patients, one with average scan time of 15 minutes and standard deviation 5 minutes, the other with average 25 and standard deviation 10. The arrival processes are approximated by Poisson processes with averages 2 and 1 per hour.
a. What is the load to this system?
b. Calculate the weighted average scan time and standard deviation.
c. Calculate the expected waiting time for the FCFS discipline and for both non-preemptive priority orders.

2. Consider a system with 4 components. All components have an exponentially distributed lifetime with expectation 1. Two components need to be up to have a functioning system. The system starts with components 3 and 4 as spare parts.
a. Calculate the expected time that the system is up when components 3 and 4 are in cold stand-by.
b. Calculate the expected time that the system is up when components 3 and 4 are in warm stand-by.
c. Calculate ϕ and Φ for this system.
d. What is the probability that the system is up after 2 time units?

3. A call center planner uses the Erlang C formula for computing the service level.
a. Give 3 aspects in which the Erlang system does not model most call centers exactly, and explain how this influences the service level.
The planner estimates the input parameters as follows: $\lambda = 10$ and $\beta = 2$. With 24 agents the probability of waiting less than 20 seconds is 0.85, according to the Erlang C formula.
b. What is the productivity?
A colleague analyses the data and says that λ is not always exactly 10, but that it can be somewhere between 9 and 11.
c. How many agents would you schedule to be sure to have approximately an 80% service level? What can you say about the productivity?
d. Explain two possible measures in many call centers that can help to deal with a λ that is not completely known, such that both the service level and the productivity are high.

4. Consider a revenue management problem with two classes of customers, type 2 books before type 1. Type 1 products cost 10, type 2 products cost 6. Total capacity is 16, and the demand for class 1 is Poisson distributed with expectation 10, the demand for class 2 can be assumed to be ∞. Type 2 purchases can be cancelled by paying a fine of 2. Type 2 customers are only cancelled where there is demand of type 1 without capacity. Consider the following simple reservation policy: as long as there is capacity type 2 bookings are made, and for every type 1 reservation a type-2 booking is cancelled.
a. Estimate the expected total revenue.
Now a booking limit for type-2 customers is used (while type-2 bookings can still be cancelled for a fine of 2).
b. Calculate the optimal booking limit.
c. Estimate again the expected total revenue.

Table with value of P(X>k) with X a Poisson distributed random variable with mean mu

	values of mu									
k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000