Exam Optimization of Business Processes
26 August 2009

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions count equally.
The use of a calculator and a dictionary are allowed.

1. A maternity ward in a small hospital has 2 beds. Arrivals are approximately Poisson, and patients who find both beds occupied go to another hospital.
a. Give a formula for the probability that the ward is full.
b. Explain why this probability is equal to the probability that new patients are blocked.
c. Let the average length of stay be 0.5 day and the demand 3 per day on average. Calculate the offered load, the blocking probability and the occupancy.

2. Consider a machine with two types of jobs. Type 1 has exponential service times with rate 2, type 2 has exponential service times with rate 3. Arrivals are according to independent Poisson processes.
a. Give the expected waiting times for both classes in the case of production in FIFO order.
b. Give the expected waiting times for both classes in the case of strict non-preemptive priority to class 1 and of non-preemptive priority to class 2. Explain the differences found.

3. A contact center has inbound calls and emails. Shifts are defined by 0-1 vectors. There are K different types of shifts, and shift k costs c_k. At interval i s_i agents are needed for inbound calls. During interval i u_i agents are required for dealing with the emails.
a. Formulate a mathematical programming model for shift scheduling during one day that minimizes costs and schedules enough agents (i.e., at least $s_i + u_i$ during interval i).
Now the emails from interval i need not necessarily be handled during interval i, but in one of the intervals $i, \ldots, i + t - 1$ for some fixed $t > 1$.
b. Formulate a mathematical programming model for shift scheduling during one day that minimizes costs and schedules enough agents.
c. Give a simple numerical example in which the answer under b is cheaper than the one under a.

4. Consider a revenue management problem with two classes of customers, type 2 books before type 1. Type 1 products cost 10, type 2 products cost 6, and type 2 purchases can be cancelled by paying a fine of 2. Type 2 customers are only cancelled where there is demand of type 1 without capacity. Total capacity is 20, and the demand for both classes is Poisson distributed with expectation 10.
a. Calculate the optimal amount of capacity to sell to type 2.
b. Calculate the expected number of type 2 customers for which the booking will be cancelled.
c. Calculate the expected total revenue.