Exam Optimization of Business Processes
25 May 2007

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1 and 4 each give 2.5 points when correctly answered, questions 2 and 3 can give 2 points.
The use of a calculator and a dictionary are allowed. A table with Poisson distributions is added.

1. A hospital has a special ambulance to transport certain types of patients. This ambulance is operational 24 hours a day, 7 days a week. The average total time that it takes to transport a patient is 90 minutes. There are on average 5 requests per day.
a. Under a homogeneous Poisson arrival process, what is the probability that an arbitrary patient has to wait?
b. In reality the arrivals are Poisson, but not homogeneous. Will the waiting probability be higher or lower than what is found under a? Motivate your answer.
c. Another hospital in the same region has a similar ambulance with the same load. What is the delay probability when both hospitals join forces? Assume again arrivals according to a homogeneous Poisson process.

2. Consider a machine with homogeneous Poisson arrivals and exponential service times.
a. Give the expectation and the variation of the waiting time of an arbitrary arriving customer.
b. Assume that the service time is uniform on $[a, b]$, $a < b$. Give the expected waiting time and explain how the variation of the waiting time can be approximated.

3. Consider a 1-out-of-2 and a 2-out-of-3 system in series.

a. Calculate ϕ and Φ for this particular system.

b. Using the expressions found under a, calculate the probability that the system is up at 10 assuming that initially all components are functioning and that the time to fail is exponentially distributed with expectation 4.

c. Now assume that there are two repairmen, one at the 1-out-of-2 system and one at the 2-out-of-3 system. The repair time is exponential with expectation 5. What is the long-run probability that the system is down?

4. Consider a revenue management problem with two classes of customers, type 2 books before type 1. Type 1 products cost 10, type 2 products cost 6, and type 2 purchases can be cancelled by paying a fine of 2. Type 2 customers are only cancelled where there is demand of type 1 without capacity. Total capacity is 20, and the demand for both classes is Poisson distributed with expectation 10.

a. Calculate the optimal amount of capacity to sell to type 2.

b. Calculate the expected number of type 2 customers for which the booking will be cancelled.

c. Calculate the expected total revenue.