Exam Optimization of Business Processes
28 August 2006

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1 and 4 each give 2 points when correctly answered, questions 2 and 3 can give 2.5 points.
The use of a calculator and a dictionary are allowed. A table with Poisson distributions is added.

1. In a hospital there are two types of patients with separate wards, both with Poisson arrival processes. Patients are admitted when there is a bed available, otherwise they are transferred to another hospital. The transfer percentage is 5% for type 1 and 40% for type 2.
a. What will happen to the transfer percentages if the two wards are merged and the same admission rule is used?
b. Describe an admission rule under which both transfer percentages decrease.
c. Describe a mathematical model by which we can estimate the transfer percentages under this admission rule.

2. Consider a machine with two types of jobs. Type 1 has exponential service times with rate 2, type 2 has exponential service times with rate 3. Arrivals are according to independent Poisson processes.
a. Give the expected waiting times for both classes in the case of production in FIFO order.
b. Give the expected waiting times for both classes in the case of strict non-preemptive priority to class 1 and of non-preemptive priority to class 2. Explain the differences found.

3. A contact center has inbound calls and emails. Shifts are defined by 0-1 vectors. There are K different types of shifts, and shift k costs c_k. At interval i s_i agents are needed for inbound calls. During interval i u_i agents are required for dealing with the emails.
a. Formulate a mathematical programming model for shift scheduling during one day that minimizes costs and schedules enough agents (i.e., at least $s_i + u_i$ during interval i).
Now the emails from interval i need not necessarily be handled during interval i, but in one of the intervals $i, \ldots, i+t-1$ for some fixed $t > 1$.
b. Formulate a mathematical programming model for shift scheduling during one day that minimizes costs and schedules enough agents.
c. Give a simple numerical example in which the answer under b is cheaper than the one under a.

4. Consider an inventory model with Poisson(5) demand, lead time 1, $K = 40$, $h = 1$, and maximal 5% backorders. Estimate Q^* and r^*.