Exam Applied Stochastic Modeling
15 February 2011

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Each question gives the same number of points when correctly answered.
The use of a calculator is allowed.
A table of the Poisson distribution is attached.

1. Consider a single server with an infinite queue to which jobs arrive according to a Poisson process with rate 1. The service time distribution has density $f(x) = 1 - \frac{1}{2}x$ if $x \in [0, 2]$, 0 otherwise. Jobs are processed in order of arrival.
a. Calculate the distribution function of the service time and draw its graph.
b. Calculate the first two moments of the service time distribution.
c. Calculate the expected long-run sojourn time in this system.

2. Consider a flight with cheap and expensive ticket (costing 100 and 300 respectively). The demand for the expensive tickets is Poisson with average 4. There are no possibilities of buying back tickets.
a. Compute the optimal number of seats that should be reserved for customers willing to buy expensive tickets.
An intermediate class with price 150 is introduced. Bookings in this class are made before the high-paying customers, demand is Poisson with average 5. Now there are two reservation levels: one for the two most expensive classes, one for the most expensive class.
b. Prove that the sum of two independent Poisson distributions has again a Poisson distribution.
c. Determine both reservation levels as to obtain the highest possible revenue. Motivate the way you calculated the reservation levels.

3. Consider a single-order inventory problem (the newsvendor problem), Poisson-distributed demand with average μ, costs for lost sales q, and costs for left-over items h.
a. Compute the optimal order level for $\mu = 5$, $q = 2$ and $h = 1$. Do the same for $\mu = 10$, $q = 2$ and $h = 1$.
b. Compare both answers and explain the findings.
c. Assume that we order thinking that $\mu = 10$, but actually $\mu = 5$. Compute the expected relative increase in costs compared to ordering when knowing that $\mu = 5$.

4. In a theme park customers arrive at a certain attraction according to a Poisson process. The attraction serves people in batches: when the kth arrives they all leaves the queue at once and start being served.
a. Model the dynamics of the waiting line as a regenerative process. Describe what the renewal points are.
b. Compute the long-run average waiting time.
c. Assume that on average 1 customer arrives per minute and that $k = 6$. Compute the probability that an arbitrary arriving customer has to wait longer than 3 minutes.