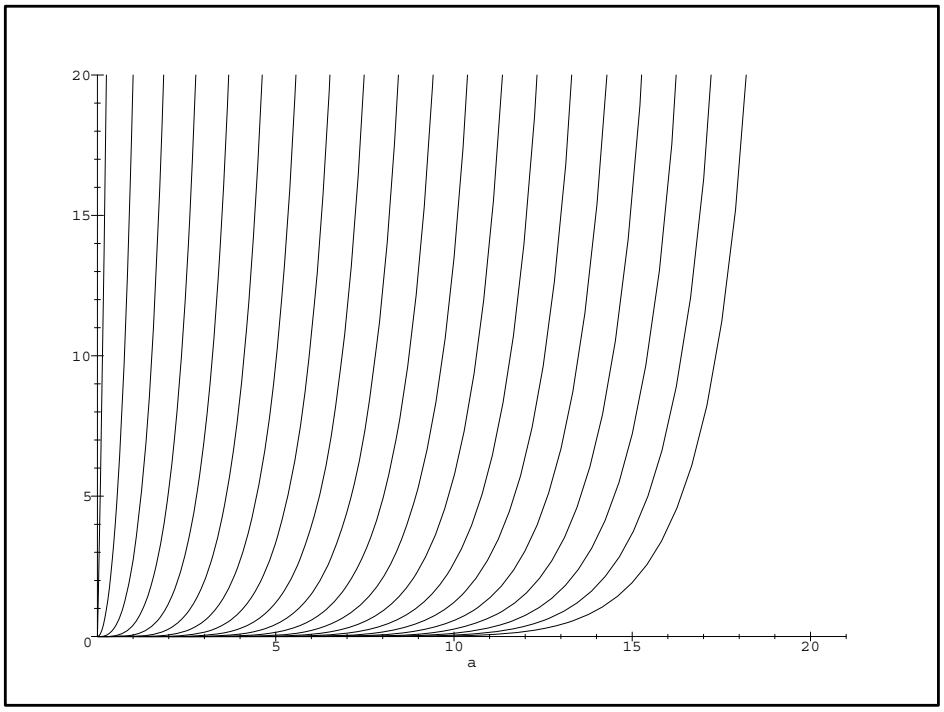
Exam Modeling of Business Processes
16 February 2005, duration: 3 hours

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 2, and 3 each give 2 points when correctly answered, question 4 can give 3 points.
The use of a calculator is allowed.
A table with the standard normal distribution is attached.

1. Consider a call center with on average 1.5 arrivals per minute, an average service time of 5 minutes, and 10 agents. The Erlang C model is used to compute the performance of this call center.
a. Compute the expected waiting time using the table on the next page.
b. Give examples of an increase in scope and an increase in scale in the context of call centers.
Suppose the call center doubles in arrival rate and in number of agents.
c. Compute the expected waiting time using the table.
d. How many agents do you need to make the average waiting time less than 90 seconds?



Values of $\mathbb{E}W_Q$ as a function of the load a for (from left to right) $s = 1$ to 20 and $\beta = 60$.

2. A petrol company is considering different designs for gas stations. One design is as follows. There are two pumps, one behind the other. If there are no cars, then an arriving car goes to pump 1. If pump 1 is occupied then an arrival goes to pump 2. If pump 2 is occupied then arrivals have to wait, even if pump 1 is not occupied. Similarly, if the car at pump 2 is ready and pump 1 is still occupied, then the car at pump 2 has to wait for the car at pump 1 to leave. For simplicity we assume Poisson arrivals and exponential service times at the gas station.

a. Assume that the pumps are available and that a car arrives. What is the time until both pumps are available again? What is the expected number of cars that will have been served by then?

b. What is the maximum arrival rate such that system is still stable?

3. A project has the following activities:

Activity	Preceding activities	Duration
A	-	2
B	A	3
C	A	2
D	C	1
E	B,D,G	2
F	-	3
G	C,F	2

Assume for the moment that there are enough resources.
a. Make a graph representation of this project.
b. Compute the earliest finish time of the project and all earliest and latest starting times of the activities. (Hint: renumber first the activities.)
c. Give the definitions of slack, critical activity, and critical path.
d. Compute in the example project the slack of each activity. What is the critical path?
Suppose that activities B and C use the same resource. Therefore they cannot be scheduled at the same time.
e. What is now the earliest finish time of the project?
f. Prove that the solution to d. gives indeed the earliest finish time possible.

4. An agricultural firm harvests K kilograms of a certain product. The company has two ways to sell their product: to Albert Heijn at a price p_r per item or at a market at a price p_m. Albert Heijn will buy all the firm is willing to sell them, the demand at the market D is random. Leftover products are worthless.
a. Formulate your expected income as a function of the amount of product that you sell to Albert Heijn.
b. Give the policy that maximizes your expected income.
c. Calculate the policy for $K = 1000$, $p_r = 0.9$, $p_m = 1.0$, and D is normally distributed with expectation 1100 and standard deviation 300.
d. The management is not only interested in maximizing expected income, but is also risk-averse. What should management do in your opinion? Explain yourself using heuristic arguments.