Exam Modeling of Business Processes
21 December 2004

This exam consists of **4** problems, each consisting of several questions.

All answers should be motivated, including calculations, formulas used, etc.

It is allowed to use 1 sheet of paper (or 2 sheets each written on one side) with **hand-written** notes.

The minimal note is 1. Each problem is divided in 3 or 5 questions, in total 16. Each question can give 0.5 points, except for questions 1a and 3a: they can give each 1 point.

The answers may be written down in English or in Dutch.

To be handed out as well: table with the Poisson distribution.

The use of a calculator and a dictionary is allowed.

1. A production line is such that at a particular machine always two parts arrive at the same time. The arrivals occur according to a Poisson process. The service times are i.i.d. exponentially distributed.

a. Give a formula for the average long-run waiting time of the first of a group of two parts that is processed.

b. Give a formula for the average long-run waiting time of an arbitrary part.

c. Compare this with the case of single-part arrivals, but with the same average number of arriving parts. Which one is bigger?

2. A part has a life time that is gamma distributed with 2 phases, which is the sum of two i.i.d. exponentially distributed random variables.

a. Give the expected time until failure of the part.

b. We consider a system with 2 parts as above in series. Give the expected time to failure.

c. Give the hazard rate of a part.

d. Give the hazard rate of the system.

e. We introduce a repairman who takes exactly d time units to repair the system. What is the average availability of the system?


3. A certain product is kept on stock. A new order is placed when the stock reaches 10, it takes 4 days for the delivery to arrive. The demand is Poisson, with an average of 2 per day. Lost sales are back ordered.

a. Calculate the expected number of backorders for an order cycle.

b. The order size is 20. Give a good approximation of the average stock level.

c. Estimate the probability that an arbitrary order needs to be back ordered.

4. A call center has two types of customer contacts: inbound calls and emails. Emails can be interrupted to take a call without loss of efficiency. During a certain interval the arrival rate λ of calls is 8 per minute, the average call duration is 2 minutes. There are 20 agents.

a. Determine the average waiting time of inbound calls using the plot below.

b. What is the expected number of emails that will be handled? They take on average 3 minutes.

It is found that on average call center agents are unproductive during 10% of their time.

c. Estimate for this situation the average waiting time of inbound calls, and determine the expected number of emails that will be handled.

d. Repeat this question for an unproductivity of 20%.

e. Are the answers you found realistic? Motivate your answers.
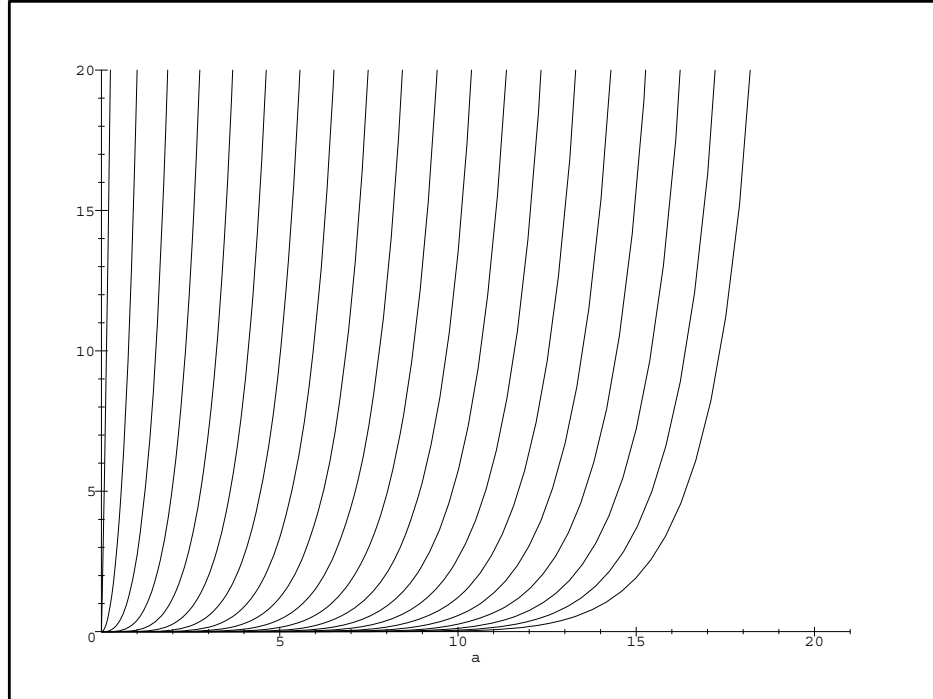
Values of $\mathbb{E}W_Q$ as a function of the load a for (from left to right) $s = 1$ t/ m 20 and $\beta = 60$.