Exam Modeling of Business Processes
17 December 2002

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Questions 1, 2, and 3 each give 2 points when correctly answered, question 4 can give 3 points.
The answers may be written down in English or in Dutch.
To be handed out as well: table with the standard normal distribution function.
The use of a calculator is allowed.

1. A call center planner uses the Erlang C formula for computing the service level.
a. Give 3 aspects in which the Erlang system does not model most call centers exactly, and explain how this influences the service level.
The planner estimates the input parameters as follows: $\lambda = 10$ and $\beta = 2$. With 24 agents the probability of waiting less than 20 seconds is 0.85, according to the Erlang C formula.
b. What is the productivity?
A colleague analyses the data and says that λ is not always exactly 10, but that it can be somewhere between 9 and 11.
c. How many agents would you schedule to be sure to have approximately an 80% service level? What can you say about the productivity?
d. Explain two possible measures in many call centers that can help to deal with a λ that is not completely known, such that both the service level and the productivity are high.

2. Consider a 2-out-of-3 system (thus a system that is up if at least 2 of its 3 components are up) with components that have independent identically distributed life times.
a. Give a closed-form expression for the availability of the system at some time t as a function of the probabilities that the components are up.
Suppose that life times are exponentially distributed.
b. What is the life time distribution of the system?
Suppose that there is a single repairman, and that repair times are also exponential.
c. Give a formula for the long-run probability that the system is up.

3. An agricultural firm harvests K kilograms of a certain product. The company has two ways to sell their product: to Albert Heijn at a price p_r per item or at a market at a price p_m. Albert Heijn will buy all the firm is willing to sell them, the demand at the market D is random. Leftover products are worthless.
a. Formulate your expected income as a function of the amount of product that you sell to Albert Heijn.
b. Give the policy that maximizes your expected income.
c. Calculate the policy for $K = 1000$, $p_r = 0.9$, $p_m = 1.0$, and D is normally distributed with expectation 1100 and standard deviation 300.
d. The management is not only interested in maximizing expected income, but is also risk-averse. What should management do in your opinion? Explain yourself using heuristic arguments.

4. A machine does two operations consecutively, each operation having an independent exponentially distributed processing time (with averages β_1 and β_2). Assume that input and output buffers can accomodate any number of parts. Orders arrive according to a Poisson process with rate λ.
a. For which parameter values is the waiting time finite?
b. Give an expression for $\mathbb{E}(X+Y)^2$ for general and independent X and Y.
c. Calculate the waiting time for $\lambda = 1$, $\beta_1 = 1/2$, and $\beta_2 = 1/3$.
Now assume that it is possible to change the machine such that the two operations can be executed at the same time.
d. Show that the service time is of the form $X + ZU + (1-Z)V$, with X, U, V, and Z independent and $Z \in \{0,1\}$.
e. Give an expression for $\mathbb{E}(X + ZU + (1-Z)V)^2$.
f. Calculate again the waiting time for $\lambda = 1$, $\beta_1 = 1/2$, and $\beta_2 = 1/3$.